MCQ Part 1

1. (Giancoli Chapter 11) A mass on a spring in SHM (Fig. 11–1) has amplitude A and period T. At what point in the motion is the velocity zero and the acceleration zero simultaneously?

   (a) $x = A$

   (b) $x > 0$ but $x < A$

   (c) $x = 0$

   (d) $x < 0$

   (e) None of the above

   Solution: The answer is **e**; a mass on a spring undergoing simple harmonic motion will never experience zero velocity and zero acceleration simultaneously.

2. (Giancoli Chapter 11) An object of mass M oscillates on the end of a spring. To double the period, replace the object with one of mass:

   (a) $2M$

   (b) $M/2$

   (c) $4M$

   (d) $M/4$

   (e) None of the above

   Solution: The answer is **c**. T is proportional to $\sqrt{M}$.

3. A grandfather clock is "losing" time because its pendulum moves too slowly. Assume that the pendulum is a massive bob at the end of a string. The motion of this pendulum can be sped up by (list all that work):

   (a) shortening the string.

   (b) lengthening the string.

   (c) increasing the mass of the bob.

   (d) decreasing the mass of the bob.

   Solution: The correct answer is **a** decreasing the length decreases the period.

4. Suppose you pull a simple pendulum to one side by an angle of 5°, let go, and measure the period of oscillation that ensues. Then you stop the oscillation, pull the pendulum to an angle of 10°, and let go. The resulting oscillation will have a period about __________ the period of the first oscillation.
   (a) four times
   (b) twice
   (c) half
   (d) one-fourth
   (e) the same as

> **Solution:** The correct answer is **e** as for the small angle approximation, the period is same regardless of the angle of displacement.

MCQ Part 2

A particle on a spring executes simple harmonic motion; when it passes through the equilibrium position it has a speed v. The particle is stopped, and then the oscillations are restarted so that it now passes through the equilibrium position with a speed of $2v$. After this change

5. the frequency of oscillation will change by a factor of
   (a) 4
   (b) $\sqrt{8}$
   (c) 2
   (d) $\sqrt{2}$
   (e) 1 (it remains unchanged)

> **Solution:** The answer is **e**. The frequency of a simple harmonic oscillator only depends on the constant k and the mass of the particle, none of which changed.

6. the maximum displacement of the particle will change by a factor of
   (a) 4
   (b) $\sqrt{8}$
   (c) 2
   (d) $\sqrt{2}$
   (e) 1 (it remains unchanged)

Solution: The answer is **c**. By conservation of energy the kinetic energy of the particle at the equilibrium point is equal to the spring potential energy of the system at maximum displacement point. This means that the amplitude is proportional to the speed at the equilibrium.

7. the magnitude of the maximum acceleration of the particle will change by a factor of
   (a) 4
   (b) $\sqrt{8}$
   (c) 2
   (d) $\sqrt{2}$
   (e) 1 (it remains unchanged)

Solution: The answer is **c** because the amplitude is doubled.

8. A particle on a spring executes simple harmonic motion. When the particle is found at $x = x_{\max}/2$ the speed of the particle is
   (a) $v_x = v_{\max}$
   (b) $v_x = \sqrt{3}v_{\max}/2$
   (c) $v_x = \sqrt{2}v_{\max}/2$
   (d) $v_x = v_{\max}/2$

Solution: The correct answer is **b**. Conservation of energy between the equilibrium point and the point of maximum displacement gives

$$x_{\max} = \sqrt{\frac{m}{k}}v_{\max}$$

Conserving energy between the equilibrium point and when $x = x_{\max}/2$ gives

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}mx^2$$

Substituting $x = x_{\max}/2$ and solving the systems gives $v_x = \sqrt{3}v_{\max}/2$

Fun Problems

9. A 5.22-kg object is attached to the bottom of a vertical spring and set vibrating. The maximum speed of the object is 15.3 cm/s and the period is 645 ms. Find (a) the force constant of the spring, (b) the amplitude of the motion, and (c) the frequency of oscillation.

Solution:

(a) The period of a vertical spring is given by $T = 2\pi\sqrt{\frac{m}{k}}$ Solving for k,

$$\boxed{k = 495\,\mathrm{N/m}}$$

(b) By conservation of energy

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

$$\boxed{A = 1.57\,\mathrm{cm}}$$

(c)

$$\boxed{f = 1.55\,\mathrm{Hz}}$$

10. A pendulum with mass m and string length l is released from rest at a height h from it's equilibrium position. What is the tension in the string at the lowest point of the pendulum's oscillation in terms of the given variables?

Solution: By conservation of energy, the speed of the pendulum at it's equilibrium position is

$$v = \sqrt{2gh}$$

The centripetal force at this point is given by

$$F_c = T - mg = \frac{mv^2}{l}$$

Plugging v into the above equation and solving for T gives

$$\boxed{T = mg(\frac{2h}{l} + 1)}$$

11. Three 10,000-kg ore cars are held at rest on a 26.0° incline on a mine railway using a cable that is parallel to the incline (Fig. 17-23). The cable is observed to stretch 14.2 cm

just before a coupling breaks, detaching one of the cars. Find (a) the frequency of the resulting oscillations of the remaining two cars and (b) the amplitude of the oscillations.

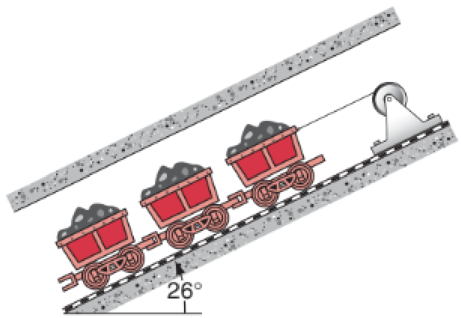

FIGURE 17-23. Exercise 15.

Solution: (a) Let one of the cars have mass m. The frequency of a simple harmonic oscillator is given by $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$. At breaking point,

$$kx = 3mg\sin\theta$$
$$k = \frac{3mg\sin\theta}{x}$$

Substituting into the first equation,

$$f = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}}$$

Plugging in 26.0° for θ and 0.142 m for x $f = 1.07$ Hz.

(b) Immediately after the third car breaks off from the rest, the equilibrium point of system changes to where the net force is zero, or the force of tension is equal to the force from the cars.

$$kx' = 2mg\sin\theta$$
$$x' = \frac{2mg\sin\theta}{k}$$

Remember from the previous problem that $k = \frac{3mg\sin\theta}{x}$ so plugging into this expression,

$$x' = \frac{2}{3}x$$

The amplitude is given by

$$\boxed{A = x - x' = 4.73\,\text{cm}}$$