MCQ Part 1

1. A solid wooden block is placed in a container of water and floats with one-third of its volume submerged. The density of the wood is most nearly:
   (a) 1000 kg/m^3
   (b) 667 kg/m^3
   (c) 500 kg/m^3
   (d) 333 kg/m^3
   (e) 100 kg/m^3

Solution: The answer is **d**. By Archimedes' principle, the buoyant force equals the weight of displaced fluid. For a floating object, $\rho_{\text{obj}}Vg = \rho_{\text{fluid}}V_{\text{sub}}g$. With $V_{\text{sub}} = V/3$:

$$\rho_{\text{obj}} = \rho_{\text{fluid}} \cdot \frac{1}{3} = \frac{1000}{3} \approx 333 \text{ kg/m}^3$$

2. A hydraulic press has a small piston of area A and a large piston of area $4A$. A force F is applied to the small piston. What force is exerted by the large piston?
   (a) $F/4$
   (b) $F/2$
   (c) F
   (d) $2F$
   (e) $4F$

Solution: The answer is **e**. Pascal's principle states that pressure is transmitted equally throughout a fluid. Since $P = F/A$ is the same at both pistons:

$$F_{\text{large}} = P \cdot 4A = \frac{F}{A} \cdot 4A = 4F$$

3. Water flows through a horizontal pipe that narrows from a large cross-sectional area to a smaller one. Compared to the wide section, the fluid in the narrow section has:
   (a) lower speed and higher pressure.
   (b) higher speed and higher pressure.
   (c) higher speed and lower pressure.
   (d) lower speed and lower pressure.

(e) the same speed and the same pressure.

Solution: The answer is **c**. By the continuity equation $A_1v_1 = A_2v_2$, a smaller cross-section means higher speed. By Bernoulli's equation, higher speed corresponds to lower pressure.

4. An object is fully submerged in a fluid and released from rest. It accelerates upward. Which of the following must be true?
   (a) The object is less dense than the fluid.
   (b) The object is more dense than the fluid.
   (c) The buoyant force equals the object's weight.
   (d) The net force on the object is zero.
   (e) The fluid pressure is uniform throughout.

Solution: The answer is **a**. If the object accelerates upward, the net force is upward, meaning the buoyant force exceeds the gravitational force. This requires $\rho_{\text{obj}} < \rho_{\text{fluid}}$.

MCQ Part 2

A large open tank is filled with water to a height H above a small hole near the bottom of the tank. Water flows out of the hole into the atmosphere. Assume the tank is large enough that the water level drops negligibly slowly.

5. The speed of the water exiting the hole is
   (a) $v = gH$
   (b) $v = \sqrt{gH}$
   (c) $v = \sqrt{2gH}$
   (d) $v = 2\sqrt{gH}$
   (e) $v = 2gH$

Solution: The answer is **c**. Applying Bernoulli's equation between the top surface and the hole (both at atmospheric pressure, top surface velocity ≈ 0):

$$P_{\text{atm}} + \rho gH + 0 = P_{\text{atm}} + 0 + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gH}$$

This is Torricelli's theorem.

6. The hole is now plugged and a second hole is drilled at a height $H/2$ above the bottom of the tank. The exit speed of water from this new hole, compared to the original, is
   (a) the same.
   (b) $\sqrt{2}$ times greater.
   (c) half as large.
   (d) $1/\sqrt{2}$ times as large.
   (e) twice as large.

> **Solution:** The answer is **d**. The new hole is at height $H/2$ from the bottom, so the effective head of water above it is $H - H/2 = H/2$. The exit speed is $v' = \sqrt{2g(H/2)} = \sqrt{gH}$, which is $1/\sqrt{2}$ times the original speed $\sqrt{2gH}$.

7. A second identical tank is filled with a fluid of density 2ρ (twice that of water) to the same height H. A hole identical in size to the original is drilled at the bottom. Compared to the water tank, the volume flow rate out of this tank is
   (a) twice as large.
   (b) $\sqrt{2}$ times as large.
   (c) the same.
   (d) $1/\sqrt{2}$ times as large.
   (e) half as large.

> **Solution:** The answer is **c**. Torricelli's theorem gives $v = \sqrt{2gH}$, which depends only on g and H, not on the fluid density. Since the hole area is the same, $Q = Av$ is identical for both tanks.

Fun Problems

8. A large cylindrical tank of cross-sectional area $A_T = 2.00\ \text{m}^2$ is filled with water to a height of $H = 3.60$ m. A circular drain hole of radius $r = 1.50$ cm is opened at the base of the tank.
   (a) Find the initial speed of the water exiting the drain.
   (b) Find the initial volume flow rate out of the tank.
   (c) Determine the initial rate at which the water level in the tank is dropping (i.e., find dH/dt at $t = 0$).

Solution:

(a) Using Torricelli's theorem:

$$v = \sqrt{2gH} = \sqrt{2(9.8)(3.60)} \approx \boxed{8.40\text{ m/s}}$$

(b) The area of the hole is $A_h = \pi r^2 = \pi(0.0150)^2 = 7.07 \times 10^{-4}\text{ m}^2$.

$$Q = A_h v = (7.07 \times 10^{-4})(8.40) \approx \boxed{5.94 \times 10^{-3}\text{ m}^3/\text{s}}$$

(c) By continuity, the flow leaving the hole must equal the rate of volume change in the tank:

$$A_T \left|\frac{dH}{dt}\right| = Q$$

$$\left|\frac{dH}{dt}\right| = \frac{Q}{A_T} = \frac{5.94 \times 10^{-3}}{2.00} \approx \boxed{2.97 \times 10^{-3}\text{ m/s}}$$

9. A U-tube manometer is open to the atmosphere on both sides. Oil of density $\rho_o = 800\text{ kg/m}^3$ is poured into the left arm until it reaches a height of $h_o = 15.0$ cm. Water ($\rho_w = 1000\text{ kg/m}^3$) is then carefully poured into the right arm.

   (a) Find the height of water h_w that must be added to the right arm so that the fluid levels in both arms are at the same height.

   (b) Now suppose extra water is poured until the water column in the right arm is $h_w = 20.0$ cm tall. By how much does the oil surface in the left arm rise above the water surface in the right arm?

Solution:

(a) At the bottom of the U-tube, the pressure from each side must be equal:

$$\rho_o g h_o = \rho_w g h_w$$

$$h_w = \frac{\rho_o}{\rho_w} h_o = \frac{800}{1000}(0.150) = \boxed{0.120\text{ m} = 12.0\text{ cm}}$$

Since $h_w < h_o$, the water surface is lower than the oil surface, which is consistent.

(b) Let the oil surface be a height Δh above the water surface. Equating pressures at the bottom:

$$\rho_o g(h_o + \Delta h/2 \cdot \ldots)$$

More cleanly: let h_o' be the new oil column height. Let x be the rise of the oil surface. By incompressibility, as the right arm rises, the left arm also rises.

Setting pressures equal at the U-tube bottom:

$$\rho_o g h'_o = \rho_w g h'_w$$

Since $h'_w = 0.200$ m:

$$h'_o = \frac{\rho_w}{\rho_o} h'_w = \frac{1000}{800}(0.200) = 0.250 \text{ m}$$

The difference in surface heights is:

$$\Delta h = h'_o - h'_w = 0.250 - 0.200 = \boxed{0.050 \text{ m} = 5.0 \text{ cm}}$$

10. A wooden raft of mass $m = 120$ kg and volume $V = 0.250$ m^3 floats on a freshwater lake ($\rho_w = 1000$ kg/m^3). A person of mass $M = 80.0$ kg steps onto the raft.
    (a) Find the fraction of the raft's volume that is submerged when the person is standing on it.
    (b) The person now holds a dense iron anchor of mass $m_a = 20.0$ kg and volume $V_a = 2.55 \times 10^{-3}$ m^3. By how much does the waterline on the raft rise compared to part (a)?
    (c) The person throws the anchor overboard. Once the anchor sinks to the bottom of the lake and the raft returns to equilibrium, is the water level of the lake higher, lower, or the same as when the anchor was on the raft? Justify your answer.

Solution:

(a) For the raft + person system to float, the buoyant force equals total weight:

$$\rho_w g V_{\text{sub}} = (m + M)g$$

$$V_{\text{sub}} = \frac{m + M}{\rho_w} = \frac{200}{1000} = 0.200 \text{ m}^3$$

Fraction submerged: $\dfrac{0.200}{0.250} = \boxed{0.800\ (80\%)}$

(b) With the anchor on the raft, the system (raft + person + anchor) floats:

$$V'_{\text{sub}} = \frac{m + M + m_a}{\rho_w} = \frac{220}{1000} = 0.220 \text{ m}^3$$

The rise in waterline on the raft $\Delta V = 0.220 - 0.200 = 0.020$ m^3. If the raft has a uniform cross-section A_{raft}, the rise is $\Delta h = \Delta V / A_{\text{raft}}$. The additional submerged volume increases by $\boxed{0.020 \text{ m}^3}$.

(c) When the anchor is on the raft, it displaces $m_a/\rho_w = 0.0200 \text{ m}^3$ of water (the raft sinks deeper by this amount). Once the anchor is thrown overboard and sinks, it only displaces its own physical volume $V_a = 2.55 \times 10^{-3} \text{ m}^3$ of water. Since $2.55 \times 10^{-3} < 0.0200 \text{ m}^3$, the total water displaced is *less* after the anchor is thrown overboard. Therefore the lake water level **drops**.