Moderate

1. (Openstax Chapter 3) A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

   Solution: $x = 130$ m, $y = 30.9$ m

2. (Openstax Chapter 3) Can a goalkeeper at her/his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

   Solution: No, the maximum range (neglecting air resistance) is about 92 m.

3. (HRK Chapter 4) Trajectories are shown for three kicked footballs. Pick the trajectory for which...

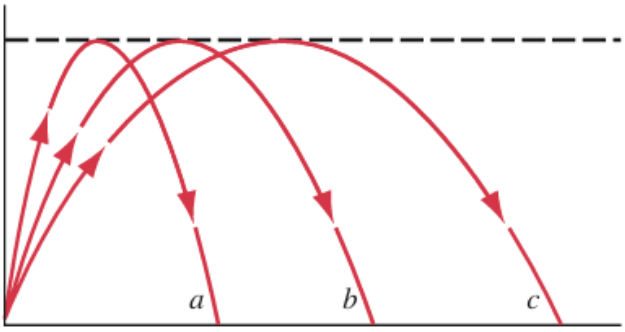
   (a) ...the time of flight is least.

   (b) ...the vertical velocity component at launch is greatest.

   (c) ...the horizontal velocity component at launch is greatest.

   (d) ...the launch speed is least.

   Ignore air resistance.



   Solution: (a) all the same (b) all the same (c) c (d) a

4. A ball rolls off the edge of a horizontal tabletop, 4.23 ft high. It strikes the floor at a point 5.11 ft horizontally away from the edge of the table.

   (a) For how long was the ball in the air?

(b) What was its speed at the instant it left the table?

Use g = 32 ft/s^2.

Solution: (a) 0.514 s (b) 9.94 ft/s

Hard

5. Prove that the maximum range for an object in projectile motion (at a constant ground level) is achieved at 45 degrees above the horizontal.

Solution: For projectile motion,

$$x = v\cos\theta t$$
$$y = v\sin\theta t - \frac{1}{2}gt^2$$

where θ is the angle above the horizontal and v is the speed that the object is launched at. The object completes its motion when y is equal to 0.

$$0 = v\sin\theta t - \frac{1}{2}gt^2$$
$$t = \frac{2v\sin\theta}{g}$$

Plugging this t value in the first equation

$$x = v\cos\theta\frac{2v\sin\theta}{g}$$
$$x = \frac{v^2 2\sin\theta\cos\theta}{g}$$

Using the trig identity $2\sin\theta\cos\theta = \sin 2\theta$

$$x = \frac{v^2\sin 2\theta}{g}$$

$\theta = 45^\circ$ maximizes $\sin 2\theta$, maximizing the horizontal range.

6. (Blue Morin) A ball is thrown with speed v at an angle θ with respect to the horizontal ground. At the highest point in the motion, the strength of gravity is somehow magically doubled. What is the total horizontal distance traveled by the ball?

Solution: $d = \frac{v_0^2 \sin\theta\cos\theta}{g}(1 + \frac{1}{\sqrt{2}})$

7. (Joshua Kim) An archer wants to hit a bullseye 77 yards away (horizontally). Unfortunately, someone has raised the target by 10.0 feet off the ground! If her bow is 3.0 feet above the ground and she releases her arrow at 50.0° with respect to the horizontal, at what velocity should he shoot her arrow? Answer in m/s.

Solution: 27 m/s