MCQ Part 1

1. A block of mass m is pushed up a frictionless ramp inclined at angle θ by a horizontal force F. The block moves up the ramp at constant velocity. Which of the following expressions correctly gives F?

   (a) $F = mg\sin\theta$

   (b) $F = mg\cos\theta$

   (c) $F = mg\tan\theta$

   (d) $F = \dfrac{mg}{\cos\theta}$

   (e) $F = \dfrac{mg}{\tan\theta}$

   Solution: The answer is **c**. Since the velocity is constant, the net force is zero. Resolving forces along and perpendicular to the ramp, the horizontal applied force has components $F\cos\theta$ along the ramp (up) and $F\sin\theta$ into the ramp (normal). Balancing along the ramp: $F\cos\theta = mg\sin\theta$, so $F = mg\tan\theta$.

2. A box of mass m sits on top of a larger box of mass M, which rests on a frictionless floor. A horizontal force F is applied to the lower box. The coefficient of static friction between the two boxes is μ_s. Which of the following is the maximum force F that can be applied to the lower box without the upper box sliding?

   (a) $F = \mu_s mg$

   (b) $F = \mu_s(m+M)g$

   (c) $F = \mu_s mg\left(\dfrac{m+M}{m}\right)$

   (d) $F = \mu_s mg\left(\dfrac{M}{m}\right)$

   (e) $F = \mu_s(m+M)g\left(\dfrac{M}{m+M}\right)$

   Solution: The answer is **c**. The only horizontal force on the upper box is friction, so its maximum acceleration is $a_{\max} = \mu_s g$. Applying Newton's Second Law to the entire two-box system: $F = (m+M)a_{\max} = \mu_s g(m+M)$. This can be rewritten as $\mu_s mg\left(\frac{m+M}{m}\right)$.

3. A skydiver of mass m falls through the air and eventually reaches terminal velocity. Which of the following correctly describes the forces on the skydiver at terminal velocity and immediately after the parachute opens?

	At terminal velocity	Immediately after chute opens
(A)	Net force = 0, $a = 0$	Net force upward, decelerating
(B)	Net force downward, $a = g$	Net force = 0, $a = 0$
(C)	Net force = 0, $a = 0$	Net force downward, still accelerating
(D)	Net force upward, decelerating	Net force = 0, $a = 0$
(E)	Net force downward, decelerating	Net force upward, decelerating

Solution: The answer is **a**. At terminal velocity the drag force equals gravity, so the net force and acceleration are both zero. When the parachute opens, drag instantly increases far above gravity, resulting in a net upward force and a deceleration (the skydiver slows down while still falling).

4. A small block is placed on a rotating turntable at a distance r from the center. The turntable spins at constant angular speed. The coefficient of static friction between the block and turntable is μ_s. If the block is moved to a distance $2r$ from the center while the angular speed is unchanged, the minimum value of μ_s required to prevent sliding changes by a factor of:

   (a) $\frac{1}{4}$

   (b) $\frac{1}{2}$

   (c) 1 (unchanged)

   (d) 2

   (e) 4

Solution: The answer is **d**. The centripetal acceleration required is $a_c = \omega^2 r$, which is provided by friction: $\mu_s mg = m\omega^2 r$, so $\mu_s = \omega^2 r/g$. Doubling r doubles the required μ_s.

MCQ Part 2

A block of mass $3m$ rests on a frictionless horizontal table and is connected by a light string over a frictionless pulley to a hanging block of mass m, as shown. A second hanging block of mass $2m$ is connected to the opposite side of the $3m$ block by another string over a second frictionless pulley. The system is released from rest. (Take downward as positive for each hanging block.)

5. What is the acceleration of the $3m$ block?

   (a) 0

(b) $\frac{g}{6}$

(c) $\frac{g}{5}$

(d) $\frac{g}{4}$

(e) $\frac{g}{3}$

Solution: The answer is **b**. The net force driving the system is $(2m - m)g = mg$ (the $2m$ block descends). The total mass being accelerated is $m + 3m + 2m = 6m$. Thus $a = mg/(6m) = g/6$.

6. What is the tension T_1 in the string connecting the $3m$ block to the m block?
   (a) $\frac{5mg}{6}$
   (b) $\frac{7mg}{6}$
   (c) mg
   (d) $\frac{7mg}{3}$
   (e) $\frac{mg}{6}$

Solution: The answer is **b**. Applying Newton's Second Law to the m block (taking upward as positive for it): $T_1 - mg = m \cdot (-g/6) \Rightarrow$ wait—the m block *rises* since $2m > m$. Taking upward as positive: $T_1 - mg = m(g/6)$, so $T_1 = mg + mg/6 = 7mg/6$.

7. If the mass of the table block is changed from $3m$ to m while the two hanging masses remain the same, the acceleration of the system will:
   (a) Remain the same.
   (b) Increase, because total mass decreased.
   (c) Remain the same, because the net driving force also changes.
   (d) Increase, because net force is unchanged but total mass is less.
   (e) Decrease, because having a lighter table block reduces friction.

Solution: The answer is **d**. The net driving force depends only on the difference in the hanging masses, which is unchanged at mg. The total accelerating mass is now $m + m + 2m = 4m$ instead of $6m$, so $a = mg/4m = g/4 > g/6$.

Free Response

8. A 12.0-kg block is on a rough horizontal surface ($\mu_s = 0.55$, $\mu_k = 0.40$). A force F is applied at an angle $\phi = 25.0°$ *below* the horizontal.
   (a) Derive an expression for the normal force on the block in terms of m, g, F, and ϕ.
   (b) Show that the threshold force needed to begin sliding is given by
   $$F_{\text{min}} = \frac{\mu_s mg}{\cos\phi - \mu_s \sin\phi}$$
   and calculate its value.
   (c) Once sliding begins, if F is kept at F_{min}, find the acceleration of the block.
   (d) Explain qualitatively why pushing downward at an angle makes it harder to slide the block compared to pushing horizontally.

Solution:

(a) Vertically: $N = mg + F\sin\phi$

(b) At the threshold of sliding, $F\cos\phi = \mu_s N = \mu_s(mg + F\sin\phi)$. Solving for F:
$$F(\cos\phi - \mu_s\sin\phi) = \mu_s mg \implies F_{\text{min}} = \frac{\mu_s mg}{\cos\phi - \mu_s\sin\phi}$$
Plugging in: $F_{\text{min}} = \frac{(0.55)(12.0)(9.8)}{\cos 25° - 0.55\sin 25°}$
$$\boxed{F_{\text{min}} = 86.1\,\text{N}}$$

(c) Kinetic friction: $f_k = \mu_k(mg + F_{\text{min}}\sin\phi) = 0.40\big[(12.0)(9.8) + 86.1\sin 25°\big] = 61.6\,\text{N}$
Net force: $F_{\text{min}}\cos 25° - f_k = 78.1 - 61.6 = 16.5\,\text{N}$
$$\boxed{a = \frac{16.5}{12.0} = 1.37\,\text{m/s}^2}$$

(d) Pushing downward increases the normal force, which increases the friction force. The larger friction means a greater applied force is required to overcome it, making it harder to slide the block.

9. A block of mass $m = 5.00\,\text{kg}$ rests on a frictionless ramp inclined at $\theta = 35.0°$. It is connected by a massless string over a frictionless pulley to a hanging mass M. The string is parallel to the incline.
   (a) Find the value of M required for the system to remain in static equilibrium.

(b) With $M = 8.00\,\text{kg}$, the system is released from rest. Derive expressions for the acceleration of the system and the tension in the string, and calculate each.

(c) If the ramp now has kinetic friction $\mu_k = 0.20$ and $M = 8.00\,\text{kg}$, how does the acceleration compare to part (b)? Calculate the new acceleration and explain the direction of the friction force on m.

Solution:

(a) Equilibrium along incline: $Mg = mg\sin\theta \Rightarrow M = 5.00\sin 35.0°$

$$\boxed{M = 2.87\,\text{kg}}$$

(b) Net force $= Mg - mg\sin\theta = (8.00 - 5.00\sin 35.0°)(9.8) = 50.2\,\text{N}$
Total mass $= 8.00 + 5.00 = 13.0\,\text{kg}$

$$\boxed{a = \frac{50.2}{13.0} = 3.86\,\text{m/s}^2}$$

$$T = M(g-a) = 8.00(9.8 - 3.86) \implies \boxed{T = 47.5\,\text{N}}$$

(c) Since M pulls the block m *up* the incline, kinetic friction acts *down* the incline on m. The friction force is $f_k = \mu_k mg\cos\theta = 0.20(5.00)(9.8)\cos 35° = 8.02\,\text{N}$.

$$a = \frac{Mg - mg\sin\theta - \mu_k mg\cos\theta}{M+m} = \frac{50.2 - 8.02}{13.0}$$

$$\boxed{a = 3.25\,\text{m/s}^2}$$

The acceleration is smaller than in (b) because friction now opposes the motion up the ramp.

10. Three blocks are stacked and connected as follows: block C (mass $3M$) sits on a frictionless floor; block B (mass $2M$) rests on top of C; block A (mass M) rests on top of B. A horizontal string connects A directly to a wall on the left. A horizontal force F is applied to C to the right. The coefficient of static friction between all block surfaces is $\mu_s = 0.45$. $M = 4.00\,\text{kg}$.

(a) Draw a free body diagram for each of the three blocks. Identify all forces and their Newton's Third Law reaction partners.

(b) Explain why block A remains stationary while B and C may accelerate.

(c) Find the maximum force F that can be applied to C without block B sliding on C.

(d) Find the tension in the string connecting A to the wall when F is at this maximum value.

Solution:

(a) *Block A*: tension T leftward (from wall string), friction from B rightward, normal from B upward, weight Mg downward.
Block B: friction from A leftward (reaction to A's rightward friction), friction from C rightward, normal forces up and down, weight $2Mg$ downward.
Block C: applied force F rightward, friction from B leftward, normal from floor upward, weight $3Mg$ downward. Newton's Third Law pairs include each friction force between surfaces (equal and opposite).

(b) Block A is tied to the wall, so any force on it is balanced by the string tension — it cannot accelerate. The string transmits the friction force from B to the wall, effectively making A a fixed surface.

(c) Because A is fixed, B and C accelerate together (assuming B doesn't slide on C). The only horizontal force on B is friction from C below (minus the reaction friction from A above). The maximum static friction from A on B is:

$$f_{AB} = \mu_s Mg = 0.45(4.00)(9.8) = 17.6\,\text{N}$$

For the B–C system, B must receive a net force to accelerate at the same a as C:

$$f_{CB} - f_{AB} = 2Ma$$

The maximum friction C can exert on B: $f_{CB,\max} = \mu_s(M+2M)g = 0.45(3)(4.00)(9.8) = 52.9\,\text{N}$
Maximum acceleration of B: $a_{\max} = \frac{52.9-17.6}{2(4.00)} = 4.41\,\text{m/s}^2$
Applying Newton's Second Law to C alone:

$$F - f_{CB} = 3Ma \Rightarrow F_{\max} = 3Ma_{\max} + f_{CB,\max} = 3(4.00)(4.41) + 52.9$$

$$\boxed{F_{\max} = 105.8\,\text{N}}$$

(d) Block A is in equilibrium: $T = f_{AB} = \mu_s Mg = 17.6\,\text{N}$ (the friction from B on A at maximum F, which equals the tension).

$$\boxed{T = 17.6\,\text{N}}$$