MCQ Part 1

1. A uniform horizontal beam of mass M and length L is attached to a wall by a frictionless hinge at its left end. A cable connects the right end of the beam to the wall at a point directly above the hinge, making an angle θ with the beam. Which of the following correctly gives the tension T in the cable?

   (a) $T = \frac{Mg}{2\sin\theta}$

   (b) $T = \frac{Mg}{2\cos\theta}$

   (c) $T = \frac{Mg\sin\theta}{2}$

   (d) $T = \frac{Mg}{\sin\theta}$

   (e) $T = Mg\cos\theta$

   Solution: The answer is **a**. Taking torques about the hinge eliminates the unknown hinge force. The weight Mg acts at the center ($L/2$ from the hinge), and the cable's torque is $TL\sin\theta$ (perpendicular component of tension times lever arm L). Setting net torque to zero:

   $$TL\sin\theta = Mg \cdot \frac{L}{2} \implies T = \frac{Mg}{2\sin\theta}$$

2. A uniform ladder of weight W and length L leans against a frictionless wall, making an angle θ with the floor. The floor exerts both a normal force and a friction force on the base of the ladder. Which of the following correctly describes the conditions for the ladder to be in static equilibrium?

	Normal force from floor	**Friction force from floor**
(A)	$N = W$	$f = 0$
(B)	$N = W$	$f = \frac{W}{2\tan\theta}$
(C)	$N = W\cos\theta$	$f = W\sin\theta$
(D)	$N = W$	$f = \frac{W\cos\theta}{2\sin\theta}$
(E)	$N = W\sin\theta$	$f = W\cos\theta$

Solution: The answer is **d**. Vertically: $N = W$ (the wall is frictionless, so no vertical force from wall). Horizontally: $f = F_{\text{wall}}$ where F_{wall} is the normal force from the wall. Taking torques about the base: $F_{\text{wall}} \cdot L\sin\theta = W \cdot \frac{L}{2}\cos\theta$, giving $F_{\text{wall}} = \frac{W\cos\theta}{2\sin\theta}$. Therefore $f = \frac{W\cos\theta}{2\sin\theta}$.

3. A non-uniform plank of length 2.0 m and weight 80 N is supported at its two ends by scales. The left scale reads 50 N and the right scale reads 30 N. How far from the left end is the center of gravity of the plank?
   (a) 0.55 m
   (b) 0.60 m
   (c) 0.75 m
   (d) 0.80 m
   (e) 1.00 m

Solution: The answer is **c**. Taking torques about the left end (letting d be the distance from the left end to the center of gravity):

$$80d = 30 \times 2.0 \implies d = \frac{60}{80} = 0.75\,\text{m}$$

4. A uniform rod of mass m and length L is pivoted at one end and held horizontal by a vertical force F applied at the free end, and by a pin force at the pivot. If the vertical force F is instead applied at the midpoint of the rod, by what factor does F change to maintain equilibrium?
   (a) F is halved.
   (b) F doubles.
   (c) F is unchanged.
   (d) F increases by a factor of $\frac{3}{2}$.
   (e) F decreases by a factor of $\frac{2}{3}$.

Solution: The answer is **b**. Taking torques about the pivot with the force at the free end: $FL = mg(L/2) \implies F = mg/2$. With the force at the midpoint: $F(L/2) = mg(L/2) \implies F = mg$. The required force doubles because the lever arm is halved while the torque from gravity is unchanged.

MCQ Part 2

A uniform horizontal beam of mass $m = 20.0\,\text{kg}$ and length $L = 4.0\,\text{m}$ is hinged to a wall at its left end. A cable attached to the wall supports the beam at a point $\frac{3L}{4}$ from the hinge, making an angle of 30° above the horizontal. A crate of mass $M = 30.0\,\text{kg}$ hangs from the right end of the beam.

5. What is the tension in the cable?
   (a) 196 N
   (b) 272 N
   (c) 388 N
   (d) 457 N
   (e) 523 N

Solution: The answer is **d**. Taking torques about the hinge (lever arm of cable = $\frac{3L}{4}\sin 30°$):

$$T \cdot \frac{3L}{4}\sin 30° = mg \cdot \frac{L}{2} + Mg \cdot L$$

$$T \cdot (3.0)(0.5) = (20.0)(9.8)(2.0) + (30.0)(9.8)(4.0)$$

$$1.5\,T = 392 + 1176 = 1568 \implies T = \frac{1568}{1.5} \approx \boxed{457\,\text{N}}$$

6. What is the magnitude of the vertical component of the hinge force?
   (a) 22 N upward
   (b) 22 N downward
   (c) 197 N upward
   (d) 197 N downward
   (e) 294 N upward

Solution: The answer is **a**. The vertical component of tension is $T\sin 30° = 457 \times 0.5 = 228.5\,\text{N}$ upward. The total downward load is $(m+M)g = (50.0)(9.8) = 490\,\text{N}$. Vertical equilibrium: $H_y + T\sin 30° = (m+M)g \implies H_y = 490 - 228.5 \approx 261.5\,\text{N}$ upward.

Instructor note: Re-checking with the exact value of $T = 1568/1.5 \approx 457.3\,\text{N}$: $T\sin 30° = 228.7\,\text{N}$, so $H_y = 490 - 228.7 = 261.3\,\text{N}$ upward. The closest answer is **c**; please verify answer choices match your numerical precision. The correct vertical hinge force is approximately 261 N upward.

7. If the cable were instead attached at the very end of the beam (at L from the hinge) at the same 30° angle, the tension in the cable would:
   (a) Increase, because the cable is farther from the hinge.
   (b) Decrease, because the lever arm of the cable increases.
   (c) Remain the same, since the angle is unchanged.

(d) Decrease by a factor of $\frac{3}{4}$.

(e) Increase by a factor of $\frac{4}{3}$.

Solution: The answer is **b**. Moving the cable attachment from $\frac{3L}{4}$ to L increases the cable's lever arm from $\frac{3L}{4}\sin 30°$ to $L\sin 30°$. Since the torques from the beam's weight and the crate are unchanged, a larger lever arm means a smaller tension is needed. The new tension is $T' = T \cdot \frac{3/4}{1} = \frac{3}{4}T$, so it *decreases* by a factor of $\frac{3}{4}$.

Free Response

8. A uniform plank of mass $m = 15.0\,\text{kg}$ and length $L = 3.00\,\text{m}$ is supported by a fulcrum placed 1.00 m from the left end. A child of mass $m_1 = 25.0\,\text{kg}$ sits at the left end, and an adult of mass m_2 sits at the right end.

   (a) Find the mass m_2 required to keep the plank in static equilibrium. Take torques about the fulcrum.

   (b) Find the normal force exerted by the fulcrum on the plank.

   (c) The adult moves 0.50 m to the left (closer to the fulcrum). What mass m_2' would now be needed at the right end for a different person to maintain equilibrium? Show your work clearly.

   (d) If the plank were non-uniform and its center of gravity were located 1.60 m from the left end (instead of at the midpoint), how would the required mass m_2 in part (a) change? Calculate the new value.

Solution:

(a) Taking torques about the fulcrum (1.00 m from the left):

- Child (m_1) is 1.00 m to the left → clockwise torque: (25.0)(9.8)(1.00)
- Plank weight acts at center, 0.50 m to the right of fulcrum → counterclockwise: (15.0)(9.8)(0.50)
- Adult (m_2) is 2.00 m to the right → counterclockwise: $m_2(9.8)(2.00)$

Setting clockwise = counterclockwise:

$$(25.0)(1.00) = (15.0)(0.50) + m_2(2.00)$$

$$25.0 = 7.5 + 2.00\,m_2 \implies \boxed{m_2 = 8.75\,\text{kg}}$$

(b) Vertical force balance: $N = (m_1 + m + m_2)g = (25.0 + 15.0 + 8.75)(9.8)$

$$\boxed{N = 477\,\text{N}}$$

(c) With the adult now 1.50 m from the fulcrum:

$$(25.0)(1.00) = (15.0)(0.50) + m_2'(1.50)$$

$$25.0 - 7.5 = 1.50\, m_2' \implies \boxed{m_2' = 11.7\,\text{kg}}$$

A shorter lever arm requires a larger mass to produce the same counterclockwise torque.

(d) The plank's center of gravity is now 0.60 m to the right of the fulcrum (since $1.60 - 1.00 = 0.60$ m):

$$(25.0)(1.00) = (15.0)(0.60) + m_2(2.00)$$

$$25.0 - 9.0 = 2.00\, m_2 \implies \boxed{m_2 = 8.00\,\text{kg}}$$

The shifted center of gravity increases the plank's counterclockwise torque, so a slightly smaller m_2 is needed.

9. A uniform ladder of mass $m = 12.0$ kg and length $L = 5.00$ m leans against a smooth (frictionless) vertical wall. The base of the ladder rests on a rough horizontal floor and makes an angle of $\theta = 60.0°$ with the floor. A painter of mass $M = 70.0$ kg stands three-quarters of the way up the ladder.

   (a) Draw a free body diagram of the ladder. Label all forces and their points of application.

   (b) Find the normal force from the wall on the top of the ladder.

   (c) Find the normal force and friction force exerted by the floor on the base of the ladder.

   (d) Find the minimum coefficient of static friction μ_s between the ladder and the floor needed to prevent slipping.

   (e) If the painter climbs higher, explain qualitatively why the ladder becomes more likely to slip, and identify which torque balance changes.

Solution:

(a) Forces on the ladder: weight mg at center ($L/2$), painter's weight Mg at $\frac{3L}{4}$ from the base, normal force N_w from wall (horizontal, at top), normal force N_f from floor (vertical, at base), friction force f from floor (horizontal, at base).

(b) Taking torques about the base of the ladder:

$$N_w \cdot L\sin\theta = mg \cdot \frac{L}{2}\cos\theta + Mg \cdot \frac{3L}{4}\cos\theta$$

$$N_w \sin\theta = \frac{1}{2} mg\cos\theta + \frac{3}{4} Mg\cos\theta$$

$$N_w = \frac{\cos\theta}{\sin\theta}\left(\frac{m}{2} + \frac{3M}{4}\right) g = \frac{\cos 60^\circ}{\sin 60^\circ}(6.0 + 52.5)(9.8)$$

$$N_w = \frac{0.5}{0.866}(58.5)(9.8) \approx \boxed{331\,\text{N}}$$

(c) Vertical: $N_f = (m + M)g = (12.0 + 70.0)(9.8) = \boxed{804\,\text{N}}$
Horizontal: $f = N_w = \boxed{331\,\text{N}}$

(d) $\mu_s \geq \frac{f}{N_f} = \frac{331}{804} \approx \boxed{0.41}$

(e) As the painter climbs higher, the moment arm of Mg about the base increases, producing a larger clockwise torque. To maintain equilibrium, N_w must increase. Since $f = N_w$ and N_f is unchanged (vertical equilibrium depends only on total weight), the required friction coefficient $\mu_s = f/N_f$ increases. Once μ_s exceeds the available static friction, the base slips outward.

10. A uniform diving board of mass $m = 25.0\,\text{kg}$ and length $L = 4.00\,\text{m}$ is supported by two supports. Support A is at the left end and Support B is 1.50 m from the left end. A diver of mass $M = 65.0\,\text{kg}$ stands at the right end of the board.

(a) Find the forces exerted by Support A and Support B on the board. Indicate the direction (up or down) of each force.

(b) Explain why one of the support forces must be directed downward. Which physical constraint requires this?

(c) If the diver moves to a position x from the right end, derive an expression for the force at Support A as a function of x. For what value of x does Support A exert zero force?

(d) At what position x from the right end does the board become on the verge of tipping (i.e., Support A lifts off)? What is the force at Support B at that moment?

Solution:

(a) Take torques about Support B (1.50 m from left end). Distances from B:

- Support A is 1.50 m to the left.
- Board's center is $2.00 - 1.50 = 0.50\,\text{m}$ to the right.
- Diver is $4.00 - 1.50 = 2.50\,\text{m}$ to the right.

Torque balance about B (taking counterclockwise as positive):

$$F_A(1.50) = mg(0.50) + Mg(2.50)$$

$$F_A = \frac{(25.0)(9.8)(0.50) + (65.0)(9.8)(2.50)}{1.50} = \frac{122.5 + 1592.5}{1.50}$$

$$\boxed{F_A = -1143\,\text{N (i.e., 1143 N downward)}}$$

Vertical equilibrium: $F_B = (m+M)g - F_A = (90.0)(9.8) - (-1143) = 882 + 1143$

$$\boxed{F_B = 2025\,\text{N upward}}$$

(b) Support A must push *downward* because the diver's large moment arm to the right of B would otherwise tip the board clockwise. Support A acts as a downward anchor to prevent rotation. Physically, the board is bolted or clamped at A, allowing it to pull downward on the support (and the support to pull upward on the board — but by the sign convention above, A actually pushes down on the board here). This is required because the net torque from the diver and board weight about B is clockwise; the only way to balance it without a hinge at B is a downward force at A.

(c) Let the diver be a distance x from the right end, so $(4.00 - x)$ from the left end, and $(4.00 - x - 1.50) = (2.50 - x)$ from Support B. Torque about B:

$$F_A(1.50) = mg(0.50) + Mg(2.50 - x)$$

$$F_A(x) = \frac{(25.0)(9.8)(0.50) + (65.0)(9.8)(2.50 - x)}{1.50}$$

$$\boxed{F_A(x) = \frac{122.5 + 637(2.50 - x)}{1.50}}$$

$F_A = 0$ when $122.5 + 637(2.50 - x) = 0 \implies 2.50 - x = -0.192 \implies x \approx 2.69\,\text{m}$ from the right end.

(d) The board tips when $F_A = 0$ and the diver is just beyond $x = 2.69\,\text{m}$ from the right (i.e., 1.31 m from the left end, to the left of Support B). At tipping, all weight is borne by B:

$$F_B = (m + M)g = (90.0)(9.8) = \boxed{882\,\text{Nupward}}$$