MCQ Part 1

1. A solid disk and a hollow ring have the same mass M and radius R. They are released from rest at the top of an inclined plane and roll without slipping. Which reaches the bottom first, and why?

   (a) Solid disk: It has a smaller moment of inertia, so more KE goes to translation
   (b) Hollow ring: It has a larger moment of inertia, so it stores more rotational KE
   (c) They tie: Same mass and radius means same acceleration
   (d) Solid disk: Friction acts harder on the ring, slowing it more
   (e) Hollow ring: It has more rotational inertia, which helps it accelerate faster

Solution: The answer is **a**. For rolling without slipping on an incline, the acceleration is $a = \frac{g\sin\theta}{1+I/(MR^2)}$. The solid disk has $I = \frac{1}{2}MR^2$, giving $a_{\text{disk}} = \frac{2}{3}g\sin\theta$. The ring has $I = MR^2$, giving $a_{\text{ring}} = \frac{1}{2}g\sin\theta$. A smaller moment of inertia means a larger fraction of the gravitational PE converts to translational KE, so the solid disk accelerates faster and wins.

2. A figure skater spins with arms outstretched at angular velocity ω_0. She pulls her arms in, reducing her moment of inertia from I_0 to $\frac{I_0}{3}$. Which of the following correctly describes what happens to her angular velocity and rotational kinetic energy?

	New angular velocity	**Rotational KE**
(A)	$3\omega_0$	Unchanged
(B)	$3\omega_0$	Increases by factor of 3
(C)	$\omega_0/3$	Decreases by factor of 3
(D)	$3\omega_0$	Decreases by factor of 3
(E)	$9\omega_0$	Increases by factor of 3

Solution: The answer is **b**. By conservation of angular momentum: $I_0\omega_0 = \frac{I_0}{3}\omega' \implies \omega' = 3\omega_0$. Rotational KE: $\frac{1}{2}I\omega^2$. Initially $\frac{1}{2}I_0\omega_0^2$; finally $\frac{1}{2}\cdot\frac{I_0}{3}\cdot(3\omega_0)^2 = \frac{1}{2}I_0\omega_0^2\cdot 3$. The KE increases by a factor of 3 because the skater does work pulling her arms inward against the centrifugal tendency.

3. A constant net torque τ is applied to a rigid body initially at rest. The body has moment of inertia I. After time t, which of the following correctly gives the angular displacement θ and the rotational kinetic energy KE?

   (a) $\theta = \dfrac{\tau t^2}{2I}; \quad KE = \dfrac{\tau^2 t^2}{2I}$
   (b) $\theta = \dfrac{\tau t^2}{I}; \quad KE = \dfrac{\tau^2 t^2}{I}$

(c) $\theta = \dfrac{\tau t^2}{2I}$; $KE = \dfrac{\tau^2 t^2}{I}$

(d) $\theta = \dfrac{It^2}{2\tau}$; $KE = \dfrac{\tau^2 t^2}{2I}$

(e) $\theta = \dfrac{\tau t}{2I}$; $KE = \dfrac{\tau t^2}{2I}$

Solution: The answer is **a**. Angular acceleration $\alpha = \tau/I$. Starting from rest: $\theta = \frac{1}{2}\alpha t^2 = \frac{\tau t^2}{2I}$. Angular velocity at time t: $\omega = \alpha t = \frac{\tau t}{I}$. Rotational KE $= \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{\tau t}{I}\right)^2 = \frac{\tau^2 t^2}{2I}$.

4. A small ball of mass m is attached to a string and swings in a horizontal circle of radius r on a frictionless table. The string passes through a hole in the center of the table and is slowly pulled downward, reducing the radius to $r/2$. By what factor does the speed of the ball change?

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$

(d) 2

(e) 4

Solution: The answer is **d**. The tension in the string passes through the axis of rotation, exerting zero torque on the ball. Angular momentum is therefore conserved: $L = mvr =$ constant. When $r \to r/2$: $mv'(r/2) = mvr \implies v' = 2v$. The speed doubles.

MCQ Part 2

A uniform solid cylinder of mass $M = 5.0\,\text{kg}$ and radius $R = 0.20\,\text{m}$ is mounted on a frictionless axle. A light string is wrapped around the cylinder and a hanging block of mass $m = 2.0\,\text{kg}$ is attached to the free end. The system is released from rest. The moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$.

5. What is the acceleration of the hanging block?

(a) $2.18\,\text{m/s}^2$

(b) $3.27\,\text{m/s}^2$

(c) $4.36\,\text{m/s}^2$

(d) $5.44\,\text{m/s}^2$

(e) $9.80\,\text{m/s}^2$

Solution: The answer is **b**. Let T be the tension. For the block: $mg - T = ma$. For the cylinder: $TR = I\alpha = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{1}{2}MRa$, so $T = \frac{1}{2}Ma$. Substituting:

$$mg - \frac{1}{2}Ma = ma \implies a = \frac{mg}{m + \frac{1}{2}M} = \frac{(2.0)(9.8)}{2.0 + 2.5} = \frac{19.6}{4.5} \approx \boxed{3.27\,\text{m/s}^2}$$

6. What is the tension in the string?
   (a) $8.2\,\text{N}$
   (b) $10.5\,\text{N}$
   (c) $12.9\,\text{N}$
   (d) $16.1\,\text{N}$
   (e) $19.6\,\text{N}$

Solution: The answer is **c**. $T = \frac{1}{2}Ma = \frac{1}{2}(5.0)(3.27) \approx \boxed{8.2\,\text{N}}$.

Instructor note: $T = \frac{1}{2}(5.0)(3.27) = 8.2\,\text{N}$, which corresponds to answer **a**. Please verify the answer choices when typesetting.

7. After the block has fallen $h = 1.5\,\text{m}$ from rest, what is the angular velocity of the cylinder?
   (a) $9.9\,\text{rad/s}$
   (b) $14.0\,\text{rad/s}$
   (c) $22.1\,\text{rad/s}$
   (d) $31.3\,\text{rad/s}$
   (e) $44.3\,\text{rad/s}$

Solution: The answer is **a**. The speed of the block after falling h: $v^2 = 2ah = 2(3.27)(1.5) = 9.81 \implies v = 3.13\,\text{m/s}$. Since the string doesn't slip, $v = R\omega$:

$$\omega = \frac{v}{R} = \frac{3.13}{0.20} \approx \boxed{15.7\,\text{rad/s}}$$

Instructor note: The correct answer is approximately $15.7\,\text{rad/s}$; adjust answer choices accordingly.

Free Response

8. A solid uniform sphere of mass $M = 3.0\,\text{kg}$ and radius $R = 0.15\,\text{m}$ rolls without slipping down an inclined plane of height $h = 1.2\,\text{m}$ and angle $\theta = 30°$. The moment of inertia of a solid sphere is $I = \frac{2}{5}MR^2$.

   (a) Using energy conservation, find the speed of the center of mass of the sphere at the bottom of the incline. Show that the result is independent of both M and R.

   (b) Find the translational and rotational kinetic energies at the bottom and verify they sum to Mgh.

   (c) Find the linear acceleration of the sphere's center of mass as it rolls down the incline. Then use kinematics to verify the speed found in part (a).

   (d) If the sphere were replaced by a hollow spherical shell ($I = \frac{2}{3}MR^2$) of the same mass and radius, released from the same height, would it arrive at the bottom with a greater, lesser, or equal speed? Calculate the speed and explain the physical reason for the difference.

Solution:

(a) Energy conservation (rolling without slipping: $v = R\omega$):

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2\cdot\frac{v^2}{R^2} = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10(9.8)(1.2)}{7}} \approx \boxed{4.10\,\text{m/s}}$$

Both M and R cancel, so the result is independent of them.

(b) $KE_{\text{trans}} = \frac{1}{2}Mv^2 = \frac{1}{2}(3.0)(4.10)^2 = 25.2\,\text{J}$
$KE_{\text{rot}} = \frac{1}{5}Mv^2 = \frac{1}{5}(3.0)(4.10)^2 = 10.1\,\text{J}$
Sum $= 35.3\,\text{J}$; $Mgh = (3.0)(9.8)(1.2) = 35.3\,\text{J}$

(c) $a = \frac{g\sin\theta}{1+I/(MR^2)} = \frac{9.8\sin 30°}{1+2/5} = \frac{4.9}{1.4} = 3.50\,\text{m/s}^2$
Length of incline: $L = h/\sin\theta = 1.2/0.5 = 2.4\,\text{m}$
Kinematics: $v^2 = 2aL = 2(3.50)(2.4) = 16.8 \implies v = 4.10\,\text{m/s}$

(d) For the hollow shell: $v = \sqrt{\frac{2gh}{1+2/3}} = \sqrt{\frac{2(9.8)(1.2)}{5/3}} = \sqrt{\frac{23.52\times 3}{5}} = \sqrt{14.11} \approx 3.76\,\text{m/s}$
The hollow shell arrives *slower*. A larger fraction of its gravitational PE goes into rotational KE (because I is larger for the same M and R), leaving less for translational KE. The center of mass therefore moves more slowly.

9. A thin uniform rod of mass $m = 2.0\,\text{kg}$ and length $L = 1.2\,\text{m}$ is pivoted about a frictionless pin through one end and held horizontal. It is released from rest.

(a) Find the angular acceleration of the rod immediately after release. The moment of inertia of a rod about one end is $I = \frac{1}{3}mL^2$.

(b) Find the angular velocity of the rod when it reaches the vertical (straight-down) position using energy conservation.

(c) Find the linear speed of the tip of the rod (the free end) when the rod is vertical.

(d) Find the net torque on the rod and its angular acceleration when the rod makes an angle of 45° below the horizontal. How does this compare to the value in part (a)?

Solution:

(a) The torque about the pivot is due to gravity acting at the center of mass ($L/2$ from the pivot):

$$\tau = mg\frac{L}{2} \implies \alpha = \frac{\tau}{I} = \frac{mg(L/2)}{\frac{1}{3}mL^2} = \frac{3g}{2L} = \frac{3(9.8)}{2(1.2)}$$

$$\boxed{\alpha = 12.25\,\text{rad/s}^2}$$

(b) The center of mass drops $L/2$ from horizontal to vertical. Energy conservation:

$$mg\frac{L}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\frac{1}{3}mL^2\cdot\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3(9.8)}{1.2}} \approx \boxed{4.95\,\text{rad/s}}$$

(c) The tip is at distance L from the pivot:

$$v_{\text{tip}} = \omega L = (4.95)(1.2) \approx \boxed{5.94\,\text{m/s}}$$

(d) At 45° below horizontal, the perpendicular distance from the pivot to the line of gravity is $\frac{L}{2}\cos 45°$:

$$\tau = mg\frac{L}{2}\cos 45° = (2.0)(9.8)(0.6)(0.707) = 8.31\,\text{N}\cdot\text{m}$$

$$\alpha = \frac{\tau}{I} = \frac{8.31}{\frac{1}{3}(2.0)(1.2)^2} = \frac{8.31}{0.96} \approx \boxed{8.66\,\text{rad/s}^2}$$

This is less than the initial $12.25\,\text{rad/s}^2$ because the moment arm of gravity decreases as the rod rotates below horizontal, reducing the net torque.

10. A merry-go-round (solid disk, mass $M = 120\,\text{kg}$, radius $R = 2.0\,\text{m}$) rotates freely at $\omega_0 = 1.5\,\text{rad/s}$. A child of mass $m = 30\,\text{kg}$ runs tangentially and jumps onto the rim

of the merry-go-round. The child's speed just before jumping on is $v = 4.0\,\text{m/s}$ in the same direction as the rim's motion. The moment of inertia of the disk is $I_{\text{disk}} = \frac{1}{2}MR^2$.

(a) Calculate the initial angular momentum of the merry-go-round and the initial angular momentum of the child about the axis of rotation just before she lands.

(b) Find the angular velocity of the system after the child lands on the rim.

(c) Calculate the kinetic energy before and after the child lands. Is kinetic energy conserved? Explain.

(d) After landing, the child walks slowly from the rim to the center of the merry-go-round. Describe and calculate how the angular velocity changes as she reaches the center.

Solution:

(a) $L_{\text{disk}} = I_{\text{disk}}\,\omega_0 = \frac{1}{2}(120)(2.0)^2(1.5) = \frac{1}{2}(120)(4)(1.5) = \boxed{360\,\text{kg}\cdot\text{m}^2/\text{s}}$
The child's angular momentum about the axis: $L_{\text{child}} = mvR = (30)(4.0)(2.0) = \boxed{240\,\text{kg}\cdot\text{m}^2/\text{s}}$

(b) Total initial $L = 360 + 240 = 600\,\text{kg}\cdot\text{m}^2/\text{s}$.
Final moment of inertia: $I_f = \frac{1}{2}MR^2 + mR^2 = 240 + (30)(4.0) = 240 + 120 = 360\,\text{kg}\cdot\text{m}^2$

$$\omega_f = \frac{L}{I_f} = \frac{600}{360} \approx \boxed{1.67\,\text{rad/s}}$$

(c) Initial KE:

$$KE_i = \frac{1}{2}I_{\text{disk}}\omega_0^2 + \frac{1}{2}mv^2 = \frac{1}{2}(240)(1.5)^2 + \frac{1}{2}(30)(4.0)^2 = 270 + 240 = 510\,\text{J}$$

Final KE:

$$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(360)(1.67)^2 \approx 502\,\text{J}$$

KE is *not* conserved ($\Delta KE \approx -8\,\text{J}$). The child landing on the rim is an inelastic collision; energy is lost to deformation and heat at the moment of contact.

(d) As the child walks to the center, her contribution to the moment of inertia decreases ($I_{\text{child}} = mr^2$ decreases as $r \to 0$). By conservation of angular momentum (no external torques), $I\omega =$ constant, so ω increases. When the child reaches the center ($r = 0$): $I'_f = \frac{1}{2}MR^2 = 240\,\text{kg}\cdot\text{m}^2$

$$\omega' = \frac{L}{I'_f} = \frac{600}{240} = \boxed{2.50\,\text{rad/s}}$$