MCQ Part 1

1. A car travels over the top of a hill of radius R. At the top of the hill, the driver experiences a normal force equal to half their weight. Which of the following correctly gives the speed of the car at the top of the hill?
   (a) $v = \sqrt{\frac{gR}{4}}$
   (b) $v = \sqrt{\frac{gR}{2}}$
   (c) $v = \sqrt{gR}$
   (d) $v = \sqrt{2gR}$
   (e) $v = \sqrt{4gR}$

2. A satellite orbits a planet of mass M at radius r with orbital period T. A second satellite orbits the same planet at radius $4r$. What is the orbital period of the second satellite?
   (a) $2T$
   (b) $4T$
   (c) $8T$
   (d) $16T$
   (e) $\sqrt{2}\,T$

3. A ball on a string moves in a vertical circle of radius R at constant speed v. At which point in the circle is the tension in the string greatest, and at which point is it least?

	Greatest tension	**Least tension**
(A)	Bottom of the circle	Top of the circle
(B)	Top of the circle	Bottom of the circle
(C)	Bottom of the circle	Side of the circle
(D)	Side of the circle	Top of the circle
(E)	Tension is the same everywhere	Tension is the same everywhere

4. Two planets, X and Y, have the same mass M but planet Y has twice the radius of planet X. An astronaut stands on the surface of each planet. Which of the following correctly compares the gravitational acceleration and the orbital speed of a satellite in low orbit?

	Surface gravity	**Low-orbit satellite speed**
(A)	$g_Y = \frac{1}{2} g_X$	$v_Y = \frac{1}{\sqrt{2}} v_X$
(B)	$g_Y = \frac{1}{4} g_X$	$v_Y = \frac{1}{\sqrt{2}} v_X$
(C)	$g_Y = \frac{1}{2} g_X$	$v_Y = \frac{1}{2} v_X$
(D)	$g_Y = \frac{1}{4} g_X$	$v_Y = \frac{1}{2} v_X$
(E)	$g_Y = 2 g_X$	$v_Y = \sqrt{2}\, v_X$

MCQ Part 2

A small block of mass m sits on the frictionless surface of a rotating horizontal turntable. The block is connected to a pin at the center of the turntable by a horizontal string of length r. The turntable rotates with a constant angular speed ω. The turntable is then gradually sped up. At some critical angular speed ω_c, the string breaks.

5. While the turntable rotates at $\omega < \omega_c$, what provides the centripetal force on the block?
   (a) The normal force from the turntable.
   (b) The weight of the block.
   (c) The tension in the string.
   (d) A fictitious outward centrifugal force.
   (e) The friction between the block and turntable.

6. The string can withstand a maximum tension $T_{\max}$. Which of the following correctly gives ω_c?
   (a) $\omega_c = \sqrt{\dfrac{T_{\max}}{mr}}$
   (b) $\omega_c = \dfrac{T_{\max}}{mr}$
   (c) $\omega_c = \sqrt{\dfrac{T_{\max} r}{m}}$
   (d) $\omega_c = \sqrt{\dfrac{mr}{T_{\max}}}$
   (e) $\omega_c = \dfrac{T_{\max}}{mr^2}$

7. If the block's mass is doubled and the string length is halved, but $T_{\max}$ is unchanged, the new critical angular speed ω_c' compared to the original ω_c is:
   (a) $\omega_c' = \dfrac{\omega_c}{2}$
   (b) $\omega_c' = \dfrac{\omega_c}{\sqrt{2}}$
   (c) $\omega_c' = \omega_c$
   (d) $\omega_c' = \sqrt{2}\,\omega_c$
   (e) $\omega_c' = 2\omega_c$

Free Response

8. A car of mass m travels at constant speed v over a circular hill of radius R and then through a circular valley of the same radius R.

(a) Derive an expression for the normal force on the car at the top of the hill. At what speed would the car become airborne at the top of the hill?

(b) Derive an expression for the normal force on the car at the bottom of the valley. A passenger of mass m_p sits in the car. Derive an expression for the apparent weight of the passenger at the bottom of the valley.

(c) The car enters the valley from the hill. If there is no friction and the car's speed at the top of the hill is v_0, use energy conservation to find the speed at the bottom of the valley (assume the hill and valley are separated by a height difference h). Then find the apparent weight of the passenger at the bottom of the valley in terms of m_p, m, g, h, v_0, and R.

(d) Suppose the radius of the valley is reduced to $R/2$ while all other quantities remain the same. Describe qualitatively and quantitatively how the apparent weight of the passenger at the bottom changes.

9. A moon of mass m orbits a planet of mass M in a circular orbit of radius r. A probe of mass $m_0 \ll M$ is launched from the surface of the planet (radius R_p) and enters a circular orbit at the same radius r as the moon.

   (a) Derive an expression for the orbital speed v of the probe in terms of G, M, and r.

   (b) Derive an expression for the orbital period T of the probe. Show that your result is consistent with Kepler's Third Law.

   (c) The probe fires its engines briefly and moves to a new circular orbit at radius $2r$. Determine the ratio of the new orbital speed to the original orbital speed, and the ratio of the new period to the original period.

   (d) The gravitational potential energy of the probe at radius r is $U = -\frac{Gm_0M}{r}$. Find the total mechanical energy (kinetic plus potential) of the probe in its original orbit at radius r, and in its new orbit at radius $2r$. Which orbit has greater total energy, and what does the sign of the total energy tell you about the orbit?

10. A conical pendulum consists of a ball of mass m attached to a string of length L. The ball moves in a horizontal circle, with the string making a constant angle θ with the vertical.

   (a) Draw a free body diagram of the ball. Identify all forces acting on it.

   (b) Derive expressions for (i) the tension T in the string, (ii) the radius r of the circular path, and (iii) the speed v of the ball, all in terms of m, g, L, and θ.

   (c) Derive an expression for the period of the conical pendulum. Show that as $\theta \to 0$, your expression reduces to the familiar formula for a simple pendulum.

   (d) A second ball of mass $2m$ is used instead. If the string length and angle θ remain the same, how does the period change? Justify your answer physically and mathematically.