MCQ Part 1

1. A solid disk and a hollow ring have the same mass M and radius R. They are released from rest at the top of an inclined plane and roll without slipping. Which reaches the bottom first, and why?

   (a) Solid disk: It has a smaller moment of inertia, so more KE goes to translation
   (b) Hollow ring: It has a larger moment of inertia, so it stores more rotational KE
   (c) They tie: Same mass and radius means same acceleration
   (d) Solid disk: Friction acts harder on the ring, slowing it more
   (e) Hollow ring: It has more rotational inertia, which helps it accelerate faster

2. A figure skater spins with arms outstretched at angular velocity ω_0. She pulls her arms in, reducing her moment of inertia from I_0 to $\frac{I_0}{3}$. Which of the following correctly describes what happens to her angular velocity and rotational kinetic energy?

	New angular velocity	**Rotational KE**
(A)	$3\omega_0$	Unchanged
(B)	$3\omega_0$	Increases by factor of 3
(C)	$\omega_0/3$	Decreases by factor of 3
(D)	$3\omega_0$	Decreases by factor of 3
(E)	$9\omega_0$	Increases by factor of 3

3. A constant net torque τ is applied to a rigid body initially at rest. The body has moment of inertia I. After time t, which of the following correctly gives the angular displacement θ and the rotational kinetic energy KE?

   (a) $\theta = \dfrac{\tau t^2}{2I}; \quad KE = \dfrac{\tau^2 t^2}{2I}$
   (b) $\theta = \dfrac{\tau t^2}{I}; \quad KE = \dfrac{\tau^2 t^2}{I}$
   (c) $\theta = \dfrac{\tau t^2}{2I}; \quad KE = \dfrac{\tau^2 t^2}{I}$
   (d) $\theta = \dfrac{I t^2}{2\tau}; \quad KE = \dfrac{\tau^2 t^2}{2I}$
   (e) $\theta = \dfrac{\tau t}{2I}; \quad KE = \dfrac{\tau t^2}{2I}$

4. A small ball of mass m is attached to a string and swings in a horizontal circle of radius r on a frictionless table. The string passes through a hole in the center of the table and is slowly pulled downward, reducing the radius to $r/2$. By what factor does the speed of the ball change?

   (a) $\frac{1}{2}$
   (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$

(d) 2

(e) 4

MCQ Part 2

A uniform solid cylinder of mass $M = 5.0\,\text{kg}$ and radius $R = 0.20\,\text{m}$ is mounted on a frictionless axle. A light string is wrapped around the cylinder and a hanging block of mass $m = 2.0\,\text{kg}$ is attached to the free end. The system is released from rest. The moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$.

5. What is the acceleration of the hanging block?
   (a) $2.18\,\text{m/s}^2$
   (b) $3.27\,\text{m/s}^2$
   (c) $4.36\,\text{m/s}^2$
   (d) $5.44\,\text{m/s}^2$
   (e) $9.80\,\text{m/s}^2$

6. What is the tension in the string?
   (a) 8.2 N
   (b) 10.5 N
   (c) 12.9 N
   (d) 16.1 N
   (e) 19.6 N

7. After the block has fallen $h = 1.5\,\text{m}$ from rest, what is the angular velocity of the cylinder?
   (a) 9.9 rad/s
   (b) 14.0 rad/s
   (c) 22.1 rad/s
   (d) 31.3 rad/s
   (e) 44.3 rad/s

Free Response

8. A solid uniform sphere of mass $M = 3.0\,\text{kg}$ and radius $R = 0.15\,\text{m}$ rolls without slipping down an inclined plane of height $h = 1.2\,\text{m}$ and angle $\theta = 30^\circ$. The moment of inertia of a solid sphere is $I = \frac{2}{5}MR^2$.

(a) Using energy conservation, find the speed of the center of mass of the sphere at the bottom of the incline. Show that the result is independent of both M and R.

(b) Find the translational and rotational kinetic energies at the bottom and verify they sum to Mgh.

(c) Find the linear acceleration of the sphere's center of mass as it rolls down the incline. Then use kinematics to verify the speed found in part (a).

(d) If the sphere were replaced by a hollow spherical shell ($I = \frac{2}{3}MR^2$) of the same mass and radius, released from the same height, would it arrive at the bottom with a greater, lesser, or equal speed? Calculate the speed and explain the physical reason for the difference.

9. A thin uniform rod of mass $m = 2.0\,\text{kg}$ and length $L = 1.2\,\text{m}$ is pivoted about a frictionless pin through one end and held horizontal. It is released from rest.

   (a) Find the angular acceleration of the rod immediately after release. The moment of inertia of a rod about one end is $I = \frac{1}{3}mL^2$.

   (b) Find the angular velocity of the rod when it reaches the vertical (straight-down) position using energy conservation.

   (c) Find the linear speed of the tip of the rod (the free end) when the rod is vertical.

   (d) Find the net torque on the rod and its angular acceleration when the rod makes an angle of 45° below the horizontal. How does this compare to the value in part (a)?

10. A merry-go-round (solid disk, mass $M = 120\,\text{kg}$, radius $R = 2.0\,\text{m}$) rotates freely at $\omega_0 = 1.5\,\text{rad/s}$. A child of mass $m = 30\,\text{kg}$ runs tangentially and jumps onto the rim of the merry-go-round. The child's speed just before jumping on is $v = 4.0\,\text{m/s}$ in the same direction as the rim's motion. The moment of inertia of the disk is $I_{\text{disk}} = \frac{1}{2}MR^2$.

    (a) Calculate the initial angular momentum of the merry-go-round and the initial angular momentum of the child about the axis of rotation just before she lands.

    (b) Find the angular velocity of the system after the child lands on the rim.

    (c) Calculate the kinetic energy before and after the child lands. Is kinetic energy conserved? Explain.

    (d) After landing, the child walks slowly from the rim to the center of the merry-go-round. Describe and calculate how the angular velocity changes as she reaches the center.