Chapter 2: Kinematics

1 1D Vectors

A **vector** is a quantity with magnitude and direction. For example, velocity is a vector that has both the speed and direction with which an object is traveling.

- A vector variable in physics is usually represented with a small arrow on top, such as $\tilde{\mathbf{r}}$
- Vectors are depicted graphically with arrows that usually represent their size

A **scalar** is a quantity with only magnitude. An example would be the measurement of time in seconds.

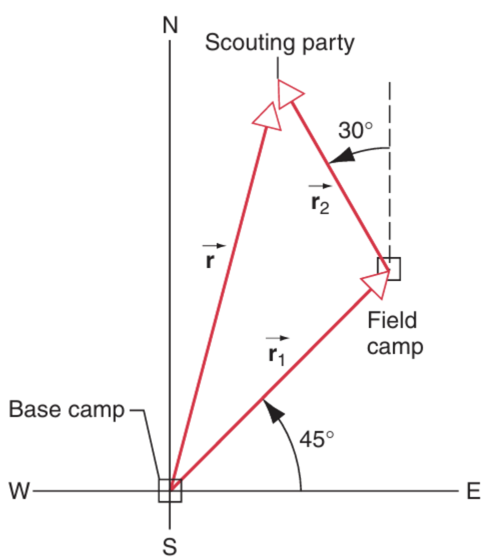

Figure 1: Example of graphically drawn vectors

As shown in this lovely example taken from the HRK textbook, each of the vectors drawn is represented by arrows indicating their respective magnitude and direction.

In Chapter 2, however, **we will only work with vectors in one dimension**. For a one-dimensional coordinate system, we usually depict left-right or up-and-down directions with + and - signs.

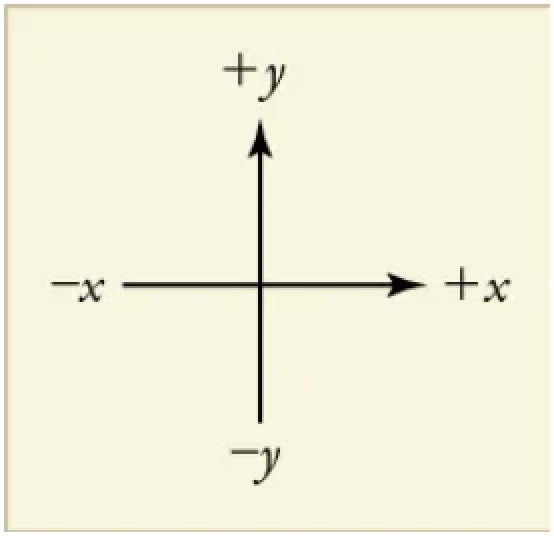

Figure 2: Right and up are usually positive, while left and down are usually negative

An example of +5 would indicate 5 units towards the right if considering only the horizontal dimension.

2 Position, Displacement, and Distance

Position is a vector. In a defined coordinate system, it is the distance from the point of origin. In Figure 2, a position of 5 units to the left has position -5 units and an arrow extending from the origin five units left.

Displacement is also a vector. It is literally the *change in position of an object.*

$$\Delta x = x_f - x_0 \tag{1}$$

where Δx is the displacement, x_f is the final position, and x_0 is the initial position.

If the object from the above example moved 2 units further left, the displacement vector that describes this motion would point from -5 units to -7 units and have a value of -2.

Distance is a scalar and is how far an object has traveled. An object that travels from position 0 to position 5 travels a distance of 5 units. However, an object that moves from 0 to 5, 5 to 0, and back from 0 to 5, travels a distance of 15 units.

Remark 1

Distance is NOT the same as displacement. Take, for instance, this example from our Openstax textbook. A professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case, her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m.

3 Time, Velocity, Speed, and Acceleration

Time is usually measured in seconds in physics.

$$\Delta t = t_f - t_0$$

The **average velocity** is defined as the change in position (displacement) divided by the change in time. Velocity is a vector:

$$\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t}$$

Remark 2

Instantaneous Velocity is the velocity of an object at a singular moment in time. For example, if one asks what is the velocity of "that" ball *at* 3 seconds, they're asking for the instantaneous velocity.

Ex. 1

Rox moves from point 0 m to position -3 m on the horizontal x axis in 3 seconds. What is her average velocity during this time interval?

Solution: Roxy has a displacement of -3 m over a change of time of 3 seconds. Here, the average velocity is simply:

$$\frac{-3 \text{ m}}{3 \text{ s}} = \boxed{-1 \frac{\text{m}}{\text{s}}}$$

Speed is a scalar quantity but is NOT just the absolute value of the velocity, although commonly referred to as such. Instead, average speed is the distance traveled over a certain amount of time.

Similarly to Velocity's relationship with displacement, **Acceleration** is the change in velocity over the change in time.

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

IMPORTANT: The slope of a position vs. time graph is the velocity, and the slope of a velocity vs time graph is the acceleration.

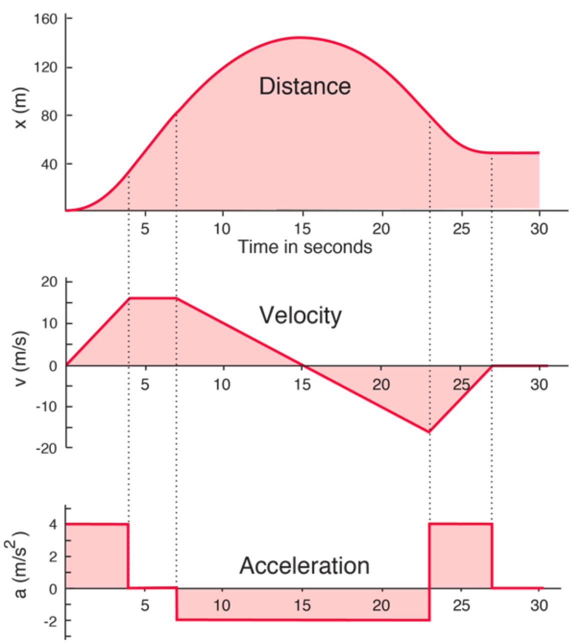

Figure 3: Position vs Time, Velocity vs Time, Acceleration vs Time, for a single object in motion

4 1D Motion Equations for Constant Acceleration

Here are the **Kinematic Equations for one dimension**. The first two are representations of position and velocity as functions of time. There is usually a fourth equation, but it is not necessary to know from my experience.

1. $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$
2. $v(t) = v_0 + at$
3. $v_f^2 = v_o^2 + 2ad$

Remark 3

Well-written proofs can be found in our Giancoli textbook, Khan Academy, or any proper looking video on youtube.

5 Practicing with Kinematic Equations

General problem solving tips

- Write using variables; don't plug in the numbers until you have the final solution - it's way faster.
- Write down what you need to solve for first and find the equation that has that quantity.
- Jot down the unknown and known quantities in complicated problems.
- Draw diagrams.
- Think about if your answer makes sense: would a person be running at 100 m/s?

Ex. 2

A ball thrown straight up takes 2.25 s to reach a height of 36.8 m. (a) What was its initial speed? (b) What is its speed at this height? (c) How much higher will the ball go?

(a) Solution: We're looking for the initial velocity, v_0. The first kinematic equation gives

$$y(t) = y_0 + v_0 t + \frac{1}{2}at^2$$

$$y(t) = v_0 t + \frac{1}{2}at^2$$

The acceleration is equal to $-g$

$$y(t) = v_0 t + \frac{1}{2}(-g)t^2$$

$$v_0 t = y(t) + \frac{1}{2}gt^2$$

$$v_0 = \frac{y(t) + \frac{1}{2}gt^2}{t}$$

Plugging in the values of $g = 9.81$ m/s^2, $t = 2.25$ s and $y(t) = 36.8$ m

$$\boxed{v_0 = 27.4\,\mathrm{m/s}}$$

(b) Solution: From the second kinematic equation

$$v_f = v_0 + at$$

$$\boxed{v_f = 5.33\,\mathrm{m/s}}$$

(c) Solution: Considering from the height of 36.8 m

$$v_f^2 = v_0^2 + 2ad$$

The velocity at the maximum height is equal to 0

$$0 = v_0^2 - 2gd$$

$$d = \frac{v^2}{2g}$$

$$\boxed{d = 1.45\,\mathrm{m}}$$